APPLICATION OF GPS FOR ASTROGATION IN LEO AND GEO

KEY TERMS

GPS

GPS - *Global Positioning System*

Consists of three segments:

- **Space segment** (satellite constellation on MEO, continuously transmitting positioning signals)
- **Ground segment** (geoinformational and surveillance stations network, providing precise orbital parameters and other data to space segment)
- **User segment** (GPS signal receivers, calculating geopositions and other data from positioning signals data)

GPS provides high precision geoposition detection (within 0.3 – 5 m for objects on the surface of the Earth). With modified positioning algorithms, it can be used for navigation on LEO.

More information is available here.

NAVIGATION

Navigation – set of practical methods allow to detect position, velocity and attitude of moving objects. Same time – practical art of usage mentioned methods in practice. In space applications equal-sense terms "*space navigation*" or "*astrogation*" are in use sometimes.

More information is available here.

PROBLEM

Build and mathematically describe the logic of application of GPS for autonomous navigation of spacecraft (SC) in Earth orbits from LEO to GEO.

Autonomous navigation of SC includes:

- Determination of current position and velocity of SC (i.e., state vectors determining the current orbit)
- Determination of current attitude of SC

Technical aspects (like design of GPS sensor, antenna parameters, SC structural design, etc.) are out of the area of this study.

GPS TECHNICAL DATA

GPS constellation includes 32 satellites in orbits with $h\approx 20180$ km and $i\approx 55°$, positioned in 6 planes with $\Delta\Omega\approx 60°$.

Satellites are equipped with nadir-oriented directed antennas with HPBW≈13°. Additionally, they are equipped with high-precision atomic clocks. These clocks are synchronized with GPS time through Ground Segment.

GPS satellites continuously transmit signals with information packages, containing:

- Precise (atomic) timestamp (UTC).
- Precise orbital state of satellite (as coefficients for ephemerides).
- Service information, including GPS satellite ID, different corrections for timestamp, ephemerides, ionosphere state, etc.

Transmission frequencies are 1575.42 MHz, 1227.60 MHz and 1176.45 MHz.

SC POSITION DETERMINATION VIA GPS

Disclaimer: in notes below I am not getting in count GPS ionosphere and troposphere corrections, because system supposed to use only in space (out of atmosphere).

POSITION DETERMINATION PROCEDURE

Process of position determination via GPS requires signals receiving from at least 4 satellites, and includes:

- Decoding messages from received signals.
- Determination of positions of GPS satellites, with algorithms converting ephemerides to state vectors in coordinate system (CS) WGS-84.
- Building of navigation solution.

GPS MESSAGE DECODING

Because of reasons mentioned in “Problem” chapter, we will not describe technical aspects of this decoding in details. They are provided in GPS official documentation. At this stage the format of GPS message is most important for the solution. Format of the message and its fields are described in paper IS-GPS-200N, chapter 6.2.9.1 (table 6-I-1).

DETERMINATION OF GPS SATELLITE POSITIONS (EPHEMERIDES CONVERSION)

Parameters required for ephemerides conversion (17 values) provided in message from GPS satellite. Conversion algorithms are provided in paper IS-GPS-200N, chapter 20.3.3.4.3 (table 20-III – 20-IV).

Conversion outputs SC state vectors $\vec{r}$ and $\vec{v}$ (and SC acceleration vector $\dot{\vec{v}}$, gathered from propagation model analytically). All output vectors are in CS WGS-84 (ECEF).

BUILDING NAVIGATION SOLUTION

There is number of methods to build a navigation solution from GPS data, including analytical solution (S. Bancroft, 1985) and numeric iterative solutions based on least square method (or Gauss-Newton method). This last family of solutions given as preferred because of its stability and reliability.

Below I describe iterative (LS/Gauss-Newton-based) method of navigation solution:

1. Introduce the concept of *pseudorange*. This is:

$$\rho_j = \sqrt{\left(x_j + x_u\right)^2 + \left(y_j + y_u\right)^2 + \left(z_j + z_u\right)^2} + c\delta t_u \qquad (1)$$

where:

ρ_j – pseudorange from receiver to satellite j (m.)

x_j, y_j, z_j – coordinates of satellite j (m., ECEF)

x_u, y_u, z_u – coordinates of receiver (m., ECEF)

δt_u – receiver clock error (s.)

c – speed of light (m.)

2. At a moment of time, GPS receiver got n signals from satellites, and made n pseudorange measurements, from what we have set of pseudoranges $\rho_1, \rho_2, \rho_3, \dots, \rho_n$. From this we need to find true state (matrix of receiver position coordinates and receiver error clock) in a form:

$$\boldsymbol{x} = \begin{bmatrix} x_u \\ y_u \\ z_u \\ c\delta t_u \end{bmatrix} \qquad (2)$$

3. To do this, first express an approximate state (matrix of coordinates and clock error) in a form:

$$\widehat{\boldsymbol{x}} = \begin{bmatrix} \widehat{x_u} \\ \widehat{y_u} \\ \widehat{z_u} \\ c\delta \widehat{t_u} \end{bmatrix} \qquad (3)$$

4. Pseudorange for this state should be calculated from state $\widehat{\boldsymbol{x}}$ (let's name it $\widehat{\rho_j}$). Relationship between true state and approximate state, in vector form, can be expressed as:

$$\boldsymbol{x_u} = \widehat{\boldsymbol{x_u}} + \boldsymbol{\Delta x_u} \quad (4)$$

5. Because we have a system of n non-linear equations, we need to linearize it to solve. To do this, expand pseudorange function to a first order Taylor series, calculate partial derivatives and simplify to:

$$\Delta\rho_j = a_{xj}\Delta x_u + a_{yj}\Delta y_u + a_{zj}\Delta z_u - c\delta t_u \quad (5)$$

6. Same time:

$$\Delta\rho_j = \widehat{\rho_j} - \rho_j \qquad (6)$$

$$a_{xj} = \frac{x_j - \widehat{x_u}}{\widehat{r_j}}, \qquad a_{yj} = \frac{y_j - \widehat{y_u}}{\widehat{r_j}}, \qquad a_{zj} = \frac{z_j - \widehat{z_u}}{\widehat{r_j}} \qquad (7,8,9)$$

$$\hat{r}_j = \sqrt{(x_j - \widehat{x_u})^2 + (y_j - \widehat{y_u})^2 + (z_j - \widehat{z_u})^2} \qquad (10)$$

7. Now we may express pseudorange equations for $\rho_1, \rho_2, \rho_3, \dots, \rho_n$ in a matrix form:

$$\boldsymbol{\Delta\rho} = \boldsymbol{H\Delta x} \qquad (11)$$

where:

$$\boldsymbol{\Delta\rho} = \begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \Delta\rho_3 \\ \dots \\ \Delta\rho_n \end{bmatrix}, \qquad \boldsymbol{H} = \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} & -1 \\ a_{x2} & a_{y2} & a_{z2} & -1 \\ a_{x3} & a_{y3} & a_{y4} & -1 \\ \dots & \dots & \dots & \dots \\ a_{xn} & a_{yn} & a_{zn} & -1 \end{bmatrix}, \qquad \boldsymbol{\Delta x} = \begin{bmatrix} \Delta x_u \\ \Delta y_u \\ \Delta z_u \\ \Delta c\delta t_u \end{bmatrix} \qquad (12,13,14)$$

8. Basing the formulas (1) – (14) generic least square solution will look like:

$$\boldsymbol{\Delta x} = (\boldsymbol{H}^T\boldsymbol{H})^{-1}\boldsymbol{H}^T\boldsymbol{\Delta\rho} \qquad (15)$$

Practical numeric solution can be done iteratively by applying calculated $\boldsymbol{\Delta x}$ into equation (4) until convergence (situation when $\boldsymbol{\Delta x}$ less than allowed error). Typically, solution converges in 3-5 iterations. *In case of noise or non-optimal geometry of pseudorange measurements weighted least squares method can be used.*

ECEF-ECI POSITION COORDINATES CONVERSION

Coordinates achieved from the state $\boldsymbol{x_u}$, are in CS WGS-84 (ECEF – Earth Centered, Earth Fixed). Their conversion to CS suitable for space applications (for example, to GCRF inertial CS) requires additional reduction, counting effects of Earth precession, nutation, form, etc.

Algorithm for such reduction can be found for example in D.A.Vallado “Fundamentals of Astrodynamics and Applications”, chapter 3.7, and also at IERS website (ITRF to GCRF conversion). Additional information is available at Wikipedia.

FILTERING

Gathered state (position vector $\vec{r}$, and time t_u calculated from $c\delta t_u$), same as all other navigation information, in any case will require filtering for practical usage. So it can (must!) be filtered in a different ways, including application of Extended Kalman Filter (EKF).

SC VELOCITY DETERMINATION VIA GPS

SC velocity can be calculated by measurement GPS signal frequency change caused by Doppler effect.

SC VELOCITY AND DOPPLER EQUATION

Classic Doppler equation describes change of frequency of electromagnetic wave in case if it is transmitted/received from/on moving objects. It can be written like:

$$f_R = f_T(1 - \frac{\overrightarrow{v_r} \cdot \vec{a}}{c}) \quad (16)$$

where:

f_R – frequency on receiver (user) side

f_T – transmission frequency on GPS satellite side

$\overrightarrow{v_r}$ – relative velocity vector of receiver and transmitter

$\vec{a}$ – unit vector of direction from receiver to transmitter

c – speed of light

Notice: in equation (16) relativistic corrections omitted on purpose because for velocity determination within navigation solution they are negligible and they are counted in GPS corrections.

Same time:

$$\overrightarrow{v_r} = \overrightarrow{v_J} - \overrightarrow{v_u} \quad (17)$$

where:

$\overrightarrow{v_J}$ – transmitter velocity vector

$\overrightarrow{v_u}$ – receiver velocity vector

Basing on formulas (16) и (17), to reduce complexity of calculations it is possible to convert frequency change into pseudorange[1]:

$$\dot{\rho}_J = -\overrightarrow{a_J}(\overrightarrow{v_u} - \overrightarrow{v_J}) + c\dot{\delta}t_u \quad (18)$$

From this, formulating in a matrix form for n satellites, we have:

$$\begin{bmatrix} \overrightarrow{a_1} & -c \\ \overrightarrow{a_2} & -c \\ \dots & -c \\ \overrightarrow{a_n} & -c \end{bmatrix} \begin{bmatrix} \overrightarrow{v_u} \\ \dot{\delta}t_u \end{bmatrix} = \begin{bmatrix} \overrightarrow{a_1}\,\overrightarrow{v_1} - \dot{\rho}_1 \\ \overrightarrow{a_2}\,\overrightarrow{v_2} - \dot{\rho}_2 \\ \dots \\ \overrightarrow{a_n}\,\overrightarrow{v_n} - \dot{\rho}_n \end{bmatrix} \quad (19)$$

With pseudoranges to ≥4 satellites, we got linear system, solvable without iterations.

FILTERING

Same as with position determination, velocity vector will be noisy, so it must be filtered, including application of EKF.

ALTERNATIVE METHODS FOR SC VELOCITY DETERMINATION

As alternative methods for SC velocity determination Initial Orbit Determination methods can be used. Especially, this can be Gibbs and Herrick-Gibbs methods, and Lambert Problem Solution. After gathering filtered series of SC positions in time, we can calculate its orbit.

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[1] More details of solution can be found in N.Sokolova, “Doppler Measurements and Velocity Estimation: Comparison of Standard and High Sensitivity Receivers”, chapter 2.3.4

Mentioned methods with detailed explanation and algorithms described for example in D.A.Vallado, “Fundamentals of Astrodynamics and Applications”, chapter 7.5.

In the context of real time autonomous navigation this methods are less promptly and less precise in counting SC maneuvers in comparison with method based on Doppler effect. Also, they are not getting in count second-order effects, for example drag force from scattered atmosphere at LEO. But despite this, in specific cases they can be preferable to Doppler-based method.

SC ATTITUDE DETERMINATION VIA GPS

Despite GPS is not purposed for such applications, its data and signal parameters (see section “GPS technical data”) allows to determine SC attitude in case of placing 2 GPS receivers on it.

Attitude determination via GPS is based on comparison of carrier phases on two antennas with different baseline vectors.

Relationship between difference of carrier phases and SC attitude can be expressed as:

$$\Delta\phi_{ijk} \approx \frac{1}{\lambda}\left(\overrightarrow{b_j} - \overrightarrow{b_i}\right) \cdot \overrightarrow{a_k} \quad (20)$$

where:

$\Delta\phi_{ijk}$ – measured difference of carrier phases of GPS signal

λ – wavelength

$\overrightarrow{b_i}, \overrightarrow{b_j}$ – baseline vectors (positions of antennas *i* and *j* on SC)

$\overrightarrow{a_k}$ - unit vector of direction from SC to transmitter on GPS satellite *k*

With this information SC attitude can be calculated with precision enough for practical usage. But this method requires resolving of phase ambiquity, and highly stable baseline.

In more details problem described in report by Frank van Graas and Michael Braasch, “GPS INTERFEROMETRIC ATTITUDE AND HEADING DETERMINATION: INITIAL FLIGHT TEST RESULTS” (available here).

USAGE OF GPS NAVIGATION IN DIFFERENT ORBITS

Because of specific design of GPS its efficiency as allothetic navigation data source can be different in different orbits.

LEO

In theory, usage of GPS in LEO is no different from, for example, aviation applications.

But in practice usage of GPS for navigation in LEO requires special algorithms (Doppler-based velocity determination, special filtering) and complex approach to navigation system implementation.

MEO И GEO

In higher orbits, GPS-based navigation system becomes less applicable. This caused by specific GPS design features, mentioned in section “GPS technical info”.

Relatively narrow transmission lobe from antennas of GPS satellite and its nadir orientation creating configuration shown on Fig.1:

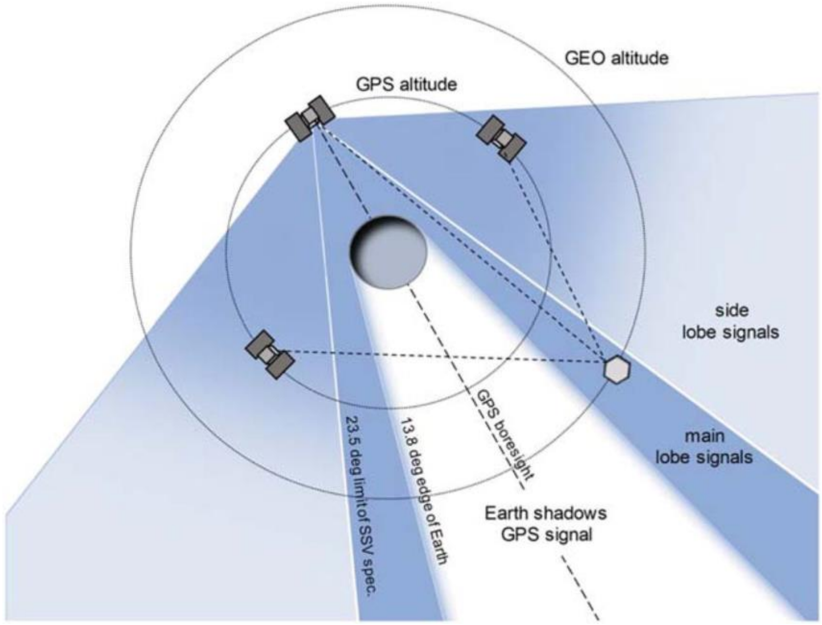

Fig.1: GPS constellation configuration as seen from HEO.

Here it is visible that from MEO/HEO SC cannot receive GPS signals clearly in number of sectors of orbit.

Main reasons of signal fading are:

- Drift SC out from GPS transmitter transmission cone
- Low sensitivity of GPS receiver
- Shadowing GPS signal by Earth.

Additionally, receiving from side lobe signals requires GPS receivers with improved sensitivity.

Summarizing, usage of GPS-based navigation system on HEO or in deep space is complicated. But such navigation system can be used as one of backup sources of navigation information. Particularly, it can be used for:

- SC velocity determination
- SC attitude determination
- As backup system for SC position determination (together with IMU, EKF and IOD methods).

RELIABILITY OF GPS-BASED SPACE NAVIGATION SYSTEM

In all mentioned cases, standalone GPS-based navigation system are not enough for reliable space navigation.

For space-grade reliability, GPS-based navigation system should work together with Inertial Measurement Unit (IMU) and star trackers (STs), and their data should be filtered by properly configured Bayesian filters (EKF, Madgwick, etc).

This way can be achieved stable work of navigation system in any Earth orbits.