IDEOLOGY AND STRATEGY OF APPLICATION OF GPS FOR ASTROGATION

PROBLEM STATEMENT

Assume that we need to implement GPS space navigation system (SNS) not as an integrated solution, but as dedicated module, able to act as standalone product.

GPS SNS REQUIREMENTS

As a standalone module, in MVP version, GPS SNS must provide for other systems of spacecraft (SC) next data:

- Navigation time
- State vectors of SC (position vector $\vec{r}$ and velocity vector $\vec{v}$)

IDEOLOGY OF GPS SNS

As a standalone module, GPS SNS must be (from logical/software point of view):

- **Universal** (for its goals) – *i.e., able to work in any space environment*
- **Algorithmically unified –** *i.e., versatility must be based on usage of environment-independent algorithms*
- **Logically autonomous** – *i.e., able to keep its state internally and be sensor-independent*
- **Configurable –** *i.e., able to be installed on different platforms and configured for different missions*
- **Robust** (with transparent quality) – *i.e., provide clear and reliable data about system precision and status*

GPS SNS IMPLEMENTATION STRATEGY

Following requirements of system ideology described above, GPS SNS must be implemented utilizing **dynamic orbit determination and estimation, based on state propagation and measurement updates via Extended Kalman Filter**.

This fulfills requirements of:

- **Versatility** (EKF is able to work with full, partial and zero measurements)
- **Algorithm unification** (EKF process prediction, propagation and update in one loop independent to input data)
- **Logical autonomy** (EKF is a closed-loop estimator, keeping internal state, covariance and time “by default”)
- **Configurability** (Update rates, noise models, measurement models - everything parameterized)
- **Robustness** (EKF is a classical method, recommended in literature; covariance matrix is quality metric by definition)

GPS SNS COORDINATE SYSTEM & UNIT AGREEMENT

Because GPS uses coordinate system (CS) WGS-84 and GPS time, SNS should operate (receive and output):

- Position and velocity vectors in CS WGS-84 (ECEF), in m and m/s
- Time as GPS time

GPS SNS CONFIGURATION SETTINGS

MANDATORY

- t_0 – initial GPS time
- $\vec{r_0}, \vec{v_0}$(in WGS-84/ECEF, at least coarse precision) – SC state at t_0
- $\boldsymbol{P}_0$ – initial covariance or standard deviations

COLD-START MODE

- Coarse SC state for t0 (slower convergence)

OPTIONAL

- Clock bias/drift initial values
- Coarse GPS satellite almanac (default data to initialize EKF before receiving fresh GPS messages)

GPS SNS INTERFACES

INPUT

- Set of decoded GPS measurements, size $0 \leq n \leq 32$, at epoch t, including
  - Position $\vec{r_j}$ and velocity $\vec{v_j}$ of GPS satellite j (calculated from ephemerides)
  - Pseudorange ρ_j
  - Doppler as pseudorange-rate $\dot{\rho}_J$

OUTPUT

- Navigation time t (GPS time)
- SC state vectors $\vec{r}$ and $\vec{v}$ (WGS-84/ECEF)
- Quality parameter (covariance matrix) $\boldsymbol{P}^+$

GPS SNS LOGIC

Following decided implementation strategy, logic of position and velocity determination in GPS SNS is typical for generic EKF logic.

*Note 1: here notation is an extension of notation given in my previous paper "**Application of GPS for astrogation in LEO and GEO**" (further – "Document 1").*

Note 2: Same as in mentioned document, minor corrections are omitted to keep logic pure and visual.

First, define EKF state vector (data inherited from previous step of EKF or from configuration parameters), in WGS-84/ECEF CS:

$$\boldsymbol{x} = \begin{bmatrix} r_x & r_y & r_z & v_x & v_y & v_z & b & \dot{b} \end{bmatrix}^T \quad (1)$$

where:

r_x, r_y, r_z – receiver position (m) from position vector $\vec{r}$

v_x, v_y, v_z – receiver velocity (m/s) from velocity vector $\vec{v}$

$b = c\delta t_u$ – clock bias (m)

$\dot{b} = c\dot{\delta} t_u$ – clock drift (m/s)

Next, define pseudorange measurement model. For GPS satellite j with known (calculated from ephemerides) position $\vec{r_j}$ (WGS-84/ECEF) at transmit time, we have:

$$\widehat{\rho_j} = \left\| \vec{r} - \vec{r_j} \right\| \quad (2)$$

$$\overrightarrow{\Delta r_j} = \vec{r} - \vec{r_j} \qquad R_j = \left\| \overrightarrow{\Delta r_j} \right\| \qquad \overrightarrow{a_j} = \frac{\overrightarrow{\Delta r_j}}{R_j} \qquad (3,4,5)$$

$$\widehat{\rho_j} = R_j + b \qquad (6)$$

From this, Jacobian row for ρ_j will be:

$$\boldsymbol{H}_{\boldsymbol{p_j}} = [a_{xj} \quad a_{yj} \quad a_{zj} \quad 0 \quad 0 \quad 0 \quad 1 \quad 0] \quad (7)$$

Now, define pseudorange-rate (Doppler) measurement model. For GPS satellite j with known (calculated from ephemerides) velocity $\vec{v_j}$ (WGS-84/ECEF) at transmit time, we have relative velocity:

$$\overrightarrow{\Delta v_j} = \vec{v} - \vec{v_j} \qquad (8)$$

This way, predicted pseudorange-rate will be:

$$\widehat{\dot{\rho}_j} = \overrightarrow{a_j}^T + \dot{b} \qquad (9)$$

Next, building Jacobian row for $\dot{\rho}_j$ we face a need to include a member of derivative w.r.t. position. It will be complex (and $\boldsymbol{I}$ here is identity matrix):

$$\frac{d\widehat{\dot{\rho}_j}}{d\vec{r}} = \frac{1}{R_j}(\boldsymbol{I} - \overrightarrow{a_j}\,\overrightarrow{a_j}^T)\,\overrightarrow{\Delta v_j} \qquad (10)$$

It may be included for accuracy, but for logical concept of the system we will omit it, and result Jacobian row will be:

$$\boldsymbol{H}_{\dot{\rho}_j} \approx [0 \quad 0 \quad 0 \quad a_{xj} \quad a_{yj} \quad a_{zj} \quad 0 \quad 1] \quad (11)$$

After this step, combine all together into EKF. First, define prediction process:

$$\boldsymbol{x}^- = f(\boldsymbol{x}^+, \Delta t) \quad (12)$$

$$\boldsymbol{P}^- = \boldsymbol{F}\boldsymbol{P}\boldsymbol{F}^T + \boldsymbol{Q} \quad (13)$$

where:

$\boldsymbol{x}^-, \boldsymbol{x}^+$ - predicted and corrected states

Δt – timestep

$\boldsymbol{P}$ - covariance matrix

$\boldsymbol{F}$ - state transition matrix (result of work of propagation model)

$\boldsymbol{Q}$ – modeled noise matrix (chosen or tuned)

Next, define EKF update process. Start with generating Jacobian corresponding to gathered measurements (using methods described above):

$$\boldsymbol{z} = [\rho_1 \quad \dots \quad \rho_n \quad \dot{\rho}_1 \quad \dots \quad \dot{\rho_n}]^T \quad (14)$$

$$\hat{\boldsymbol{z}} = h(\boldsymbol{x}^-) \qquad (15)$$

$$\boldsymbol{H} = \frac{dh}{dx}\bigg|_{x^-} \qquad (16)$$

where:

$\hat{\boldsymbol{z}}$ - predicted measurements (pseudorange, pseudorange-rate)

h - methods of building Jacobian rows (described above)

$\boldsymbol{H}$ - full Jacobian of state

Next, calculate residual:

$$\boldsymbol{y} = \boldsymbol{z} - \hat{\boldsymbol{z}} \qquad (17)$$

Then, generate Kalman gain:

$$\boldsymbol{K} = \boldsymbol{P}^- \boldsymbol{H}^T (\boldsymbol{H}\boldsymbol{P}^- \boldsymbol{H}^T + \boldsymbol{R})^{-1} \qquad (18)$$

where:

$\boldsymbol{K}$ – Kalman gain

$\boldsymbol{R}$ – measured noise matrix

After this, finalizing EKF update by making correction:

$$\boldsymbol{x}^+ = \boldsymbol{x}^- + \boldsymbol{K}\boldsymbol{y} \qquad (19)$$

$$\boldsymbol{P}^+ = (\boldsymbol{I} - \boldsymbol{K}\boldsymbol{H})\boldsymbol{P}^- \qquad (20)$$

From here, we naturally achieving new state $\boldsymbol{x}^+$, representing SC state vectors $\vec{r}$ and $\vec{v}$ and clock bias, and state covariance matrix $\boldsymbol{P}^+$ (used as quality metric).

GPS SNS WORK ALGORITHM

GPS SNS work algorithm in verbal form can be presented like this:

INITIALIZATION

- *If configuration settings exists:*
  - *Load given configuration settings*
- *Else load settings for cold start*

UPDATE

- *Collect and decode measurements from GPS satellites (if available)*
- *Apply EKF filter*
  - *Predict EKF state*
    - *Load modeled noise*
    - *Calculate predicted state and covariance matrix*
  - *Update EKF state*
    - *Load actual measurements (if available)*
    - *Calculate predicted measurements*
    - *Calculate Jacobian of state*
    - *Calculate residuals*
    - *Calculate Kalman gain*
  - *Correct EKF state*
    - *Calculate corrected state and covariance matrix*

PRACTICAL USAGE OF GPS SNS

APPLICABILITY DOMAIN

GPS SNS is applicable from LEO to GEO and even beyond as long as:

- Sufficient GPS signal strength is available
- Measurement geometry is not degenerate

PRECISION DEGRADATION ON WEAK SIGNAL AND GEOMETRY

With increasing orbit altitude, the number of usable GPS measurements will decrease because of reasons described in Document 1.

In such conditions, GPS SNS will face next issues:

- EKF update frequency will decrease
- Navigation solution will rely more on state propagation than on actual measurements
- Uncertainty of position determination will grow

Despite this, velocity determination will stay observable via Doppler measurements.

On very high altitudes Doppler measurements (pseudorange-rate) will become the dominant source of navigation information for the GPS SNS.

Important, that EKF will naturally reflect precision degradation via covariance growth in corresponding state components.

PRECISION CONTROL

The covariance matrix $\boldsymbol{P}^{+}$ is the primary indicator of the quality of navigation solution.

But very important to understand, that no fixed thresholds can be assumed because covariance levels are mission-dependent.

COLD START CASE

Cold start initialization will lead to slower convergence and larger initial covariance. In this case very important to provide sustained measurement availability to let EKF reach nominal accuracy ASAP.

SAFETY USAGE NOTE

GPS SNS can be used totally standalone, but it is not universal navigation solution, and cannot replace all navigation systems in all mission phases.